STUDENT'S NAME: ______________________________

TEACHER'S NAME: ______________________________

HURLSTONE AGRICULTURAL HIGH SCHOOL

2020 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics -Extension 2

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions in Section II, show all relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6 – 12)

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks

Attempt Questions 1 – 10.

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

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1. Which expression is equal to $\int 2x\, e^{-2x}\, dx$?

   (A) $-xe^{-2x} + \int e^{-2x}\, dx$

   (B) $-xe^{-2x} - \int e^{-2x}\, dx$

   (C) $-2xe^{-2x} + \int e^{-2x}\, dx$

   (D) $-2xe^{-2x} - \int e^{-2x}\, dx$

2. A student wants to prove that there is an infinite number of prime numbers. To prove this statement by contradiction, what assumption would the student start their proof with?

   (A) There is only one prime number that is even.

   (B) There is an infinite number of Primes.

   (C) There is a finite number of Primes.

   (D) All prime numbers are less than 100

3. Which of the following is the complex number $4\sqrt{3} - 4i$?

   (A) $4e^{-\frac{i\pi}{6}}$

   (B) $4e^{\frac{5\pi}{6}}$

   (C) $8e^{-\frac{i\pi}{6}}$

   (D) $8e^{\frac{5\pi}{6}}$

4. A particle is describing SHM in a straight line with an amplitude of 4 metres. Its speed is 6m/s when the particle is 2 metres from the centre of the motion.

   What is the period of the motion?

   (A) $\frac{\sqrt{3}\pi}{2}$

   (B) $\frac{2\sqrt{3}\pi}{3}$

   (C) $\sqrt{3}\pi$

   (D) $\frac{2\sqrt{2}\pi}{3}$

5. If $\underset{\sim}{u} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix}$ is a non zero vector, then the corresponding unit vector is:

(A) $\hat{\underset{\sim}{u}} = \begin{pmatrix} \frac{1}{6} \\ -\frac{1}{3} \\ \frac{1}{6} \end{pmatrix}$

(B) $\hat{\underset{\sim}{u}} = \begin{pmatrix} \frac{2}{3} \\ \frac{-1}{3} \\ \frac{2}{3} \end{pmatrix}$

(C) $\hat{\underset{\sim}{u}} = \begin{pmatrix} 1 \\ -\frac{1}{2} \\ 1 \end{pmatrix}$

(D) $\hat{\underset{\sim}{u}} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

6. A particle moves in simple harmonic motion along the *x*-axis about the origin. Initially, the particle is at its extreme positive position. The amplitude of the motion is 12 metres and the particle returns to its initial position every 3 seconds.

What is the equation for the position of the particle at time *t* seconds?

(A) $x = 12\cos\dfrac{2\pi t}{3}$

(B) $x = 24\cos\dfrac{2\pi t}{3}$

(C) $x = 12\cos 3t$

(D) $x = 24\cos 3t$

7. Which vector is perpendicular to $\begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$?

(A) $\begin{pmatrix} 6 \\ -4 \\ 8 \end{pmatrix}$

(B) $\begin{pmatrix} -3 \\ 2 \\ -4 \end{pmatrix}$

(C) $\begin{pmatrix} 3 \\ 6 \\ 1 \end{pmatrix}$

(D) $\begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix}$

8. A particle is moving along a straight line. At time t, its velocity is v and its displacement from a fixed origin is x.

If $\frac{dv}{dx} = \frac{1}{2v}$, which of the following best describes the particle's acceleration and velocity?

(A) constant acceleration and constant velocity.

(B) constant acceleration and decreasig velocity.

(C) constant acceleration and increasing velocity.

(D) increasing acceleration and increasing velocity

9. If $\frac{5}{(2x+1)(2-x)} = \frac{A}{2x+1} + \frac{B}{2-x}$, then A and B have values of:

(A) $A = -1, B = 2$

(B) $A = 1, B = -2$

(C) $A = 2, B = -1$

(D) $A = 2, B = 1$

10. The equation, in Cartesian form, of the locus of the point z if $|z + 2i| = |z + 4|$ is:

(A) $x - 2y + 3 = 0$

(B) $2x - y + 3 = 0$

(C) $x + 2y + 3 = 0$

(D) $2x + y + 3 = 0$

End of Section I

Section II

90 marks

Attempt Questions 11 – 16.

Allow about 2 hours and 45 minutes for this section.

Answer each question in a separate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a separate writing booklet.

(a) If $a = 5 + 3i$ and $b = 3 - 4i$, evaluate the following:

(i) ab **1**

(ii) $\frac{a}{b}$ **1**

(iii) $\sqrt{b}$ **2**

(b) Let $z = \sqrt{3} + i$

(i) Express z in modulus-argument form. **2**

(ii) Find the smallest positive integer n such that $z^n - \left(\overline{z}\right)^n = 0$ **3**

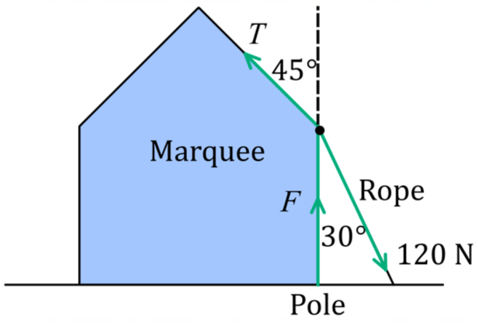
(c) The polynomial $P(x) = x^3 - 5x^2 + ax + b$, where a and b are real, has one root at $x = 3 - 2\sqrt{2}i$.

(i) Solve the equation $P(x) = 0$ **2**

(ii) Hence find the values of a and b. **1**

Question 11 continues on page 6

(d) The side of a marquee is supported by a vertical pole supplying a force of F newtons and a rope with a tension of 120 newtons. The tension in the marquee fabric is T newtons as shown below.



Not to scale

By resolving forces horizontally and vertically, or otherwise, find the exact values of T and F. **3**

End of Question 11

Question 12 (15 marks) Use a separate writing booklet.

(a) Evaluate:

(i) $\int \sin^3 x \cos^2 x \, dx$. **2**

(ii) $\int \frac{dx}{\sqrt{6 + 4x - x^2}}$. **3**

(b) (i) If $\frac{4x + 10}{(2 - x)(x^2 + 2)} = \frac{A}{2 - x} + \frac{Bx + C}{x^2 + 2}$, find the values of *A*, *B* and *C*. **3**

(ii) Hence evaluate $\int \frac{(4x + 10)\, dx}{(2 - x)(x^2 + 2)}$. **3**

(c) A point *P*, which moves in the complex plane, is represented by the equation $|z - (4 + 3i)| = 5$.

(i) Sketch the locus of the point *P*. **1**

(ii) Find the value of arg z when *P* is in the position that maximises $|z|$. **1**

(iii) Find the modulus of *z* when $\arg z = \tan^{-1}\left(\frac{1}{3}\right)$. **2**

End of Question 12

Question 13 (15 marks) Use a separate writing booklet.

(a) If $z = \sqrt{2} - \sqrt{6}i$,

(i) Express z in modulus-argument form. **2**

(ii) Evaluate z^3. **1**

(b) Find the point(s) of intersection of the line with parametric equation **4**

$$r = i + 3j - 4k + t(i + 2j + 2k)$$

and the sphere with equation

$$(x-1)^2 + (y-3)^2 + (z+4)^2 = 81.$$

(c) For d, an integer where $d > 1$,

(i) Show that $\dfrac{1}{d^2} < \dfrac{1}{d(d-1)}$ **1**

(ii) Noting that $\dfrac{1}{d^2-d} = \dfrac{1}{d-1} - \dfrac{1}{d}$ show that, for a positive integer n : **3**

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots\dots\dots + \frac{1}{n^2} < 2.$$

(d) A mass has acceleration a ms^{-2} given by $a = v^2 - 3$, where v ms^{-1} is the velocity of the mass when it has a displacement of x metres from the origin. Find v in terms of x given that $v = -2$ where $x = 1$. **4**

End of Question 13

Question 14 (15 marks) Use a separate writing booklet.

(a) (i) By considering the cases where a positive integer k is even ($k = 2x$) and odd ($k = 2x + 1$), show that $k^2 + k$ is always even. **2**

(ii) Using the result in part (i), prove, by mathematical induction, that for all positive integral values of n, $n^3 + 5n$ is divisible by 6. **3**

(b) For two positive real numbers a and b, prove that their arithmetic mean $\frac{a+b}{2}$ is always greater than or equal to their geometric mean $\sqrt{ab}$. **2**

(c) Consider two lines. l_1 and l_2, with vector equations $\underset{\sim}{r}_1$ and $\underset{\sim}{r}_2$ respectively.

(i) Find $\underset{\sim}{r}_1$, the vector equation of l_1, in the direction of $\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ and passing through **1**

The point $(-1, 2, -3)$.

The line l_2 has the vector equation $\underset{\sim}{r}_2 = (-t+1)\underset{\sim}{i} + (2t-2)\underset{\sim}{j} + (3t+6)\underset{\sim}{k}$ where $t \in \mathbb{R}$.

(ii) Find a vector parallel to l_2. **1**

(iii) Find the point of intersection of l_1 and l_2. **3**

(iv) Find the acute angle between l_1 and l_2. **3**

Give your answer in degrees correct to one decimal place.

End of Question 14

Question 15 (15 marks) Use a separate writing booklet.

(a) (i) Use De Moivre's Theorem to express $\cos 5\theta$ and $\sin 5\theta$ in terms of powers of $\sin\theta$ and $\cos\theta$. **2**

(ii) Write an expression for $\tan 5\theta$ in terms of t, where $t = \tan\theta$. **1**

(iii) By solving $\tan 5\theta = 0$, deduce that: $\tan\frac{\pi}{5}\tan\frac{2\pi}{5}\tan\frac{3\pi}{5}\tan\frac{4\pi}{5} = 5$. **3**

(b) (i) Show that $f(x) = \frac{2+x^2}{4-x^2}$ can be written as $f(x) = -1 + \frac{6}{4-x^2}$ **1**

(ii) Find the exact area enclosed by the graph of $f(x) = \frac{2+x^2}{4-x^2}$ **3**

the x-axis, and the lines $x = -1$ and $x = 1$.

(c) Consider two complex numbers, u and v, such that $\text{Im}(u) = 2$ and $\text{Re}(v) = 1$. **2**

Given that $u+v=-uv$, find the values of u and v.

(d) A subset of the complex plane is described by the relation $\text{Arg}(z-2i)=\frac{\pi}{6}$.

(i) Find the Cartesian equation of this relation. **2**

(ii) Draw a sketch of this relation. **1**

End of Question 15

Question 16 (15 marks) Use a separate writing booklet.

(a) The coordinates of three points are $A=\begin{pmatrix}1\\0\\5\end{pmatrix}, B=\begin{pmatrix}-1\\2\\4\end{pmatrix}, C=\begin{pmatrix}3\\5\\2\end{pmatrix}$

Prove that $\angle ABC$ is a right angle. **2**

(b) Prove that $33^n - 16^n - 28^n + 11^n$ is divisible by 85 for all positive integers $n \geq 2$. **3**

(c) If $P = i + j + k$ and $R = 9i + 3j + 8k$, find the point Q on $\overrightarrow{PR}$ such that $PQ : QR = 2 : 3$. **3**

(d) Let $I_n = \int_0^1 x^n \tan^{-1} x \, dx$ where $n = 0, 1, 2, \ldots$

(i) Show that $(n+1) I_n = \frac{\pi}{4} - \int_0^1 \frac{x^{n+1}}{1+x^2} dx$ for $n \geq 0$. **2**

(ii) Hence, or otherwise, find the value of I_0. **1**

(iii) Show that $(n+3) I_{n+2} + (n+1) I_n = \frac{\pi}{2} - \frac{1}{n+2}$ **2**

(iv) Hence find the value of I_4. **2**

End of Paper

2020 Trial HSC Examination

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}, \ r \neq 1$$

$$S = \frac{a}{1-r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

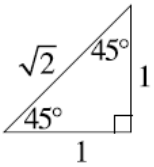
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

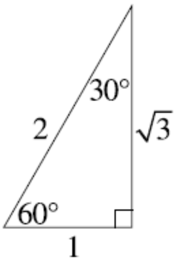
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

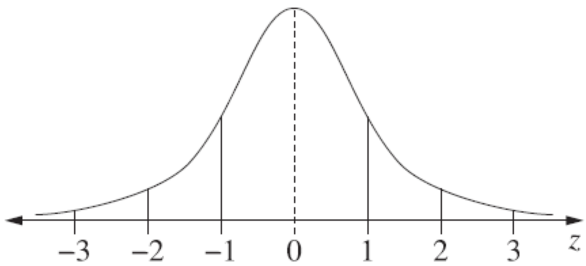
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1-p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x) = \binom{n}{x}p^x(1-p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx} dx = uv - \int v\frac{du}{dx} dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2+y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^2(x-c)$$

Hurlstone Agricultural High School

2020 Trial Higher School Certificate Examination
Mathematics Extension 2

Name ____________________ Teacher ____________________

Section I – Multiple Choice Answer Sheet

Allow about 25 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

HAHS Maths Extension 2 Trial Exam 2020

Marking Gudelines.

Outcomes Addressed in this Paper:

MEX12-2 chooses appropriate strategies to construct arguments and proofs in both practical and abstract settings.

MEX12-3 uses vectors to model and solve problems in two and three dimensions.

MEX12-4 uses the relationship between algebraic and geometric representations of complex numbers and complex number techniques to prove results, model and solve problems.

MEX12-5 applies techniques of integration to structured and unstructured problems.

MEX12-6 uses mechanics to model and solve practical problems.

Section I: Multiple Choice:

No	Working	Answer
1	$\int 2x\, e^{-2x}\, dx$ $u = 2x \qquad v' = e^{-2x}$ $u' = 2 \qquad v = -\frac{1}{2}\, e^{-2x}$ $uv - \int vu'$ $2x\left(-\frac{1}{2}\, e^{-2x}\right) - \int \left(-\frac{1}{2}\, e^{-2x}\right)(2)$ $= -\, xe^{-2x} + \int e^{-2x}\, dx$	**A**
2	Contradicting an infinite number of primes is that there is a finite number of primes	**C**
3	$4\sqrt{3} - 4i$ in 4th quadrant therefore angle is $-\frac{\pi}{6}$ $e^{-\frac{i\pi}{6}} = cos\left(-\frac{\pi}{6}\right) + isin\left(-\frac{\pi}{6}\right)$ $= \frac{\sqrt{3}}{2} - \frac{i}{2}$ *Need to multiply by 8 to give desired result.* $8e^{-\frac{i\pi}{6}} = 8\left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right) = 4\sqrt{3} - 4i$	**C**
4	Using $v^2 = n^2(a^2 - x^2)$ $6^2 = n^2(4^2 - 2^2)$ $36 = 12n^2$ $n^2 = 3$ *i.e.* $n = \sqrt{3}$ Periodic Time = $\frac{2\pi}{n} = \frac{2\pi}{\sqrt{3}} = \frac{2\sqrt{3}\pi}{3}$	**B**

5	$$|u| = \left|\begin{pmatrix}4\\-2\\4\end{pmatrix}\right| = \sqrt{4^2 + (-2)^2 + 4^2} = \sqrt{36} = 6$$ $$\hat{u} = \begin{pmatrix}\frac{4}{6}\\ \frac{-2}{6}\\ \frac{4}{6}\end{pmatrix} = \begin{pmatrix}\frac{2}{3}\\ \frac{-1}{3}\\ \frac{2}{3}\end{pmatrix}$$	**B**

Question 6 **A**

The period is 3 seconds.

$3 = \frac{2\pi}{n} \Rightarrow n = \frac{2\pi}{3}$

When $t = 0$, $x = 12$.

So the equation of motion is $x = 12\cos\frac{2\pi t}{3}$.

7	Show that the dot product is zero. Test each option and find only D works. $2 \times 3 + 5 \times (-2) + 1 \times 4 = 6 - 10 + 4 = 0$ Therefore option D is perpendicular	**D**

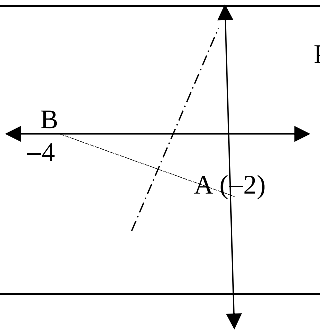
Q8:

Question 9 **C**

$\frac{dv}{dx} = \frac{1}{2v}$ and so $v\frac{dv}{dx} = a = v\left(\frac{1}{2v}\right) = \frac{1}{2}$. Therefore, the acceleration is constant.

Since the acceleration is also positive, the velocity is increasing.

9	$$\frac{5}{(2x+1)(2-x)} = \frac{A}{2x+1} + \frac{B}{2-x}$$ $\mathbf{5 = A(2-x) + B(2x+1)}$ **When $x = 2$** $5 = 5B$ $\therefore B = 1$ **When $x = -\frac{1}{2}$** $5 = \left(2\frac{1}{2}\right)A$ $A = 2$ ***i.e* A = 2, B = 1**	**D**

10	 $\lvert z + 2i\rvert = \lvert z + 4\rvert$ Perpendicular bisector of AB Gradient AB = −½ ∴ Gradient Locus = 2 Midpoint = (−2, −1) $y - -1 = 2(x - -2)$ $y + 1 = 2x + 4$ $\mathbf{2x - y + 3 = 0}$	**B**

Year 12 Examination (task 4)	Extension 2 Mathematics	2020 Trial HSC
Question No. 11	**Solutions and Marking Guidelines**	

HSC Outcome	**Solutions**	**Marking Guidelines**
MEX12-4	Question 11 (a) (i) $ab=(5+3i)(3-4i)$ $=15-20i+9i-12i^2$ $=15-11i+12$ $=27-11i$	Award 1 ~complete correct solution
MEX12-4	Question 11 (a)(ii) $\frac{a}{b}=\frac{5+3i}{3-4i}\times\frac{3+4i}{3+4i}$ $=\frac{(5+3i)(3+4i)}{9-16i^2}$ $=\frac{15+20i+9i+12i^2}{9+16}$ $=\frac{15+29i-12}{25}$ $=\frac{3+29i}{25}\quad or\quad \frac{3}{25}+\frac{29i}{25}\quad or\quad \frac{3}{25}+\frac{29}{25}i$	Award 1 ~complete correct solution
MEX12-4	Question 11 (a)(iii) $\sqrt{b}=\sqrt{3-4i}$ let $\sqrt{3-4i}=x+yi$ (where x and y are real numbers) $3-4i=(x+yi)^2$ $=x^2-y^2+2xy$	Award 2 ~complete correct solution Award 1 ~significant progress towards correct solution

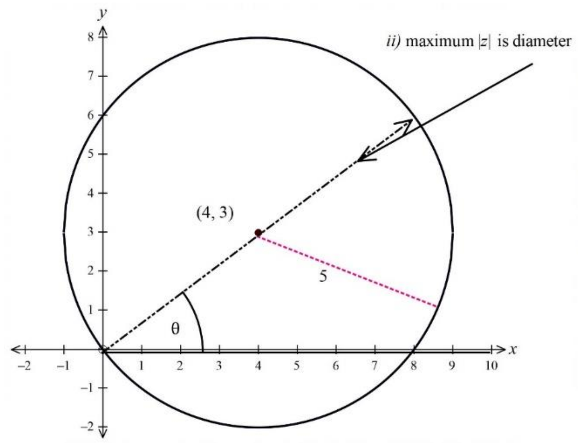
	equating real and imaginary parts gives: $x^2 - y^2 = 3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(A)$ $2xy = -4 \Rightarrow xy = 2 \Rightarrow x = \frac{-2}{y} \ldots\ldots(B)$ Substitute (B) into (A) $\left(\frac{-2}{y}\right)^2 - y^2 = 3$ $\frac{4}{y^2} - y^2 = 3y^2$ $\therefore y^4 + 3y^2 - 4 = 0$ $\therefore y^2 = \frac{-3 \pm \sqrt{9+16}}{2}$ $= \frac{-3 \pm 5}{2}$ $= 1$ or -4 (since y is a real number, $y^2 > 0$, hence, $y^2 = -4$ is not a solution) $\therefore y = \pm 1 \quad \therefore x = \mp 2$ and $\sqrt{3-4i} = \pm(2-i)$	
MEX12-4	Question 11 (b)(i) $z = \sqrt{3} + i$ $\|z\| = \sqrt{(\sqrt{3})^2 + (1)^2} = 2$ $Arg\, z = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$ $\therefore z = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ or $2cis\frac{\pi}{6}$ in modulus-argument form	Award 2 ~complete correct solution Award 1 ~significant progress towards correct solution

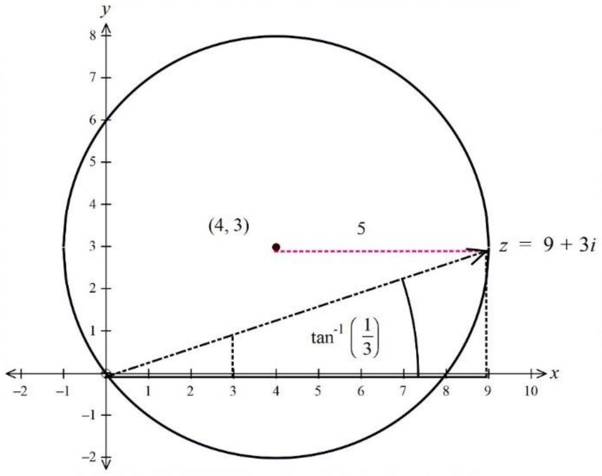
MEX12-4	Question 11 (b)(ii) $z^n - \left(\bar{z}\right)^n = 0$ $\left(2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\right)^n - \left(2\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)\right)^n = 0$ Using De Moivre's Theorem $2^n\left(\cos\frac{n\pi}{6} + i\sin\frac{n\pi}{6}\right) - 2n\left(\cos\frac{n\pi}{6} - i\sin\frac{n\pi}{6}\right) = 0$ $\cos\frac{n\pi}{6} i\sin\frac{n\pi}{6} - \cos\frac{n\pi}{6} + i\sin\frac{n\pi}{6} = 0$ $2i\sin\frac{n\pi}{6} = 0$ $\frac{n\pi}{6} = k\pi$ $\therefore n = 6k$ Smallest positive integer of n occurs when $k = 1$, $\therefore n = 6$	Award 3 ~Complete correct solution Award 2 ~Significant progress towards correct solution Award 1 ~Limited progress towards correct solution
MEX12-4	Question 11 (c)(i) $P(x) = 0$ let α, β and γ be the roots. $\alpha = 3 - 2\sqrt{2}i$ (given) $\beta = 3 + 2\sqrt{2}i$ (conjugate of α is a root) Use sum of roots to find γ $3 + 2\sqrt{2}i + 3 - 2\sqrt{2}i + \gamma = \frac{-(-5)}{1}$ $6 + \gamma = 5$ $\therefore \gamma = -1$ $\therefore$ Solution for $P(x) = 0$ is $x = 3 - 2\sqrt{2}i, 3 + 2\sqrt{2}i, -1$	Award 2 ~complete correct solution Award 1 ~significant progress towards correct solution

MEX12-4	Question 11 (c)(ii) $a=\gamma\alpha+\gamma\beta+\alpha\beta$ $\therefore a=-1\left(3-2\sqrt{2}i\right)+(-1)\left(3+2\sqrt{2}i\right)+\left(3-2\sqrt{2}i\right)\left(3+2\sqrt{2}i\right)$ $=-3+2\sqrt{2}i-3-2\sqrt{2}i+9+8$ $=11$ $b=-(\alpha\beta\gamma)$ $=-(-1)\left(3-2\sqrt{2}i\right)\left(3+2\sqrt{2}i\right)$ $=1(9+8)$ $=17$ $\therefore a=11$ and $b=17.$	Award 1 ~correct answer
MEX12-6	Question 11 (d) Resolving horizontal forces $-T\sin 45°+120\sin 30°=0$ $T\sin 45°=60$ $T=60\sqrt{2}$ Resolving verticle forces $T\cos 45°+F-120\cos 30°=0$ $60+F-60\sqrt{3}=0$ $F=60\sqrt{3}-60$ $\therefore$ Solution is $T=60\sqrt{2},\ F=60\sqrt{3}-60$	Award 3 ~Complete correct solution Award 2 ~Significant progress towards correct solution Award 1 ~Limited progress towards correct solution

Year 12	Mathematics Extension 2	Assess. Task 4 2020 HSC
Question No. 12	Solutions and Marking Guidelines	

Part / Outcome	**Solutions**	**Marking Guidelines**
(a)(i) **MEX12-5**	$\int \sin^3 x\cos^2 x\,dx = \int \sin x\left(1-\cos^2 x\right)\cos^2 x\,dx$ $= -\int\left(1-u^2\right)u^2\,du$ let $u=\cos x$, $du=-\sin x\,dx$ $=\int\left(u^4-u^2\right)du$ $=\frac{u^5}{5}-\frac{u^3}{3}+C$ $=\frac{\cos^5 x}{5}-\frac{\cos^3 x}{3}+C$ 76	<u>2 marks</u> : correct solution <u>1 mark</u> : substantially correct solution
(a)(ii) **MEX12-5**	$\int\frac{dx}{\sqrt{6+4x-x^2}}=\int\frac{dx}{\sqrt{10-(x^2-4x+4)}}$ $=\int\frac{dx}{\sqrt{\left(\sqrt{10}\right)^2-\left(x-2\right)^2}}$ $=\sin^{-1}\frac{x-2}{\sqrt{10}}+C$	<u>3 marks</u> : correct solution <u>2 marks</u> : substantially correct solution <u>1 mark</u> : partially correct solution
(b)(i) **MEX12-5**	$\frac{4x+10}{\left(2-x\right)\left(x^2+2\right)}=\frac{A}{2-x}+\frac{Bx+C}{x^2+2}$ $4x+10=A\left(x^2+2\right)+\left(Bx+C\right)\left(2-x\right)$ $8+10=A\left(4+2\right)+0$ when $x=2$ ie $A=3$ $10=2A+2C$ when $x=0$ $10=6+2C$ $C=2$ $B=3$ $\frac{4x+10}{\left(2-x\right)\left(x^2+2\right)}=\frac{3}{2-x}+\frac{3x+2}{x^2+2}$	<u>3 marks</u> : correct solution <u>2 marks</u> : substantially correct solution <u>1 mark</u> : partially correct solution

(b)(ii) **MEX12-5**	$$\int \frac{4x+10}{(2-x)(x^2+2)}dx = \int \left(\frac{3}{2-x} + \frac{3x+2}{x^2+2} \right) dx$$ $$= -3\int \frac{-1}{2-x}dx + \frac{3}{2}\int \frac{2x}{x^2+2}dx + 2\int \frac{1}{x^2+2}dx$$ $$= -3\ln(2-x) + \frac{3}{2}\ln(x^2+2) + \frac{2}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$$	<u>3 marks</u> : correct solution <u>2 marks</u> : substantially correct solution <u>1 mark</u> : partially correct solution
(c)(i) **MEX12-4**	□□ $\lvert z - (4+3i) \rvert = 5$ is a circle centre (4, 3) radius 5  	<u>1 mark</u> : correct solution *(NB: circle passes through the origin, and this **must** be shown – this is the classic 3,4,5 situation you should be aware of !* *- also, it's key to answering (ii) and (iii))*
(c)(ii) **MEX12-4**	Maximum value of $\lvert z \rvert$ is when z lies along the diameter, opposite origin So since this passes through the centre (4, 3), $$\arg z = q = \tan^{-1}\left(\frac{3}{4}\right)$$	<u>1 mark</u> : correct solution
(c)(iii) **MEX12-4**	When $\arg z = \tan^{-1}\left(\frac{1}{3}\right)$, continuing the ray to the circle, gives z = 9 + 3i.	<u>2 marks</u> : correct solution <u>1 mark</u> : substantially correct solution



So $|z| = \sqrt{9^2 + 3^2} = \sqrt{90} = 3\sqrt{10}$

Alternately the point z can be found algebraically:

z lies on $(x - 4)^2 + (y - 3)^2 = 25$

z also lies on $\quad y = \frac{x}{3}$ since $\arg z = \tan^{-1}\left(\frac{1}{3}\right)$

$$(x - 4)^2 + \left(\tfrac{x}{3} - 3\right)^2 = 25$$
$$10x^2 - 90x = 0$$
$$x = 0 \text{ or } 9$$
$$\text{so } x = 9 \text{ and } y = 3$$
$$\text{ie } z = 9 + 3i$$

Note:

Making the (common) assumption that $z = \tan^{-1}\left(\frac{1}{3}\right)$ implies $z = 3 + i$, does not show the depth of understanding this question required.

In fact, it is more a demonstration of incorrectly applying rote learning without paying any regard to the actual situation.

No marks were awarded in this case, as it does not fit the category of "*substantially correct*"

Question 13:

Outcome		Marking Guidelines
(a)MEX 12-4	(i) $Z = \sqrt{2} - \sqrt{6}i$ $r = \sqrt{(\sqrt{2})^2 + (\sqrt{6})^2}$ $\tan\theta = \frac{-\sqrt{6}}{\sqrt{2}}$ $= \sqrt{2+6}$ $\tan\theta = -\sqrt{3}$ $= \sqrt{8}$ $\theta = -\frac{\pi}{3}$ $= 2\sqrt{2}$ $\therefore z = 2\sqrt{2}\left(cos\left(-\frac{\pi}{3}\right) + i\,sin\left(-\frac{\pi}{3}\right)\right)$	2 marks for correct modulus and argument 1 mark for significant working toward modulus and argument
(a)	(ii) $z^3 = \left[2\sqrt{2}\,cis\left(-\frac{\pi}{3}\right)\right]^3$ $= 16\sqrt{2}\,cis\;3\left(-\frac{\pi}{3}\right)$ $= 16\sqrt{2}\,cis\,(-\pi)$ $= -16\sqrt{2}$	1 mark for correct answer

(b) MEX 12-3	$r = i + 3j - 4k + t(i + 2j + 2k)$ $x = 1 + t$ $y = 3 + 2t$ $z = -4 + 2t$ Now $(x-1)^2 + (y-3)^2 + (z+4)^2 = 81$ $(1+t-1)^2 + (3+2t-3)^2 + (-4+2t+4)^2 = 81$ $(t)^2 + (2t)^2 + (2t)^2 = 81$ $9t^2 = 81$ $t^2 = 9$ $t = \pm 3$ $\therefore$Points are: $[1+3, 3+2(3), -4+2(3)] = (4, 9, 2)$ $[1-3, 3+2(-3), -4+2(-3)] = (-2, -3, -10)$	4 marks for two correct points 3 marks for substantial progress toward solving the equations simultaneously or equivalent merit 2 marks for writing parametric equations with some progress toward answer or equivalent merit 1 mark for writing parametric equations or other initial or other limited working relevant to question

(c)MEX 12-2	(i) If $d > 1$ then $d > d - 1$ Also $d^2 > d(d-1)$ $\therefore$ $\frac{1}{d^2} < \frac{1}{d(d-1)}$	1 mark for correct solution

	(ii) Given $\frac{1}{d(d-1)} = \frac{1}{d-1} - \frac{1}{d}$ From (i) $\frac{1}{d^2} < \frac{1}{d-1} - \frac{1}{d}$ For $\frac{1}{1^2} < \frac{1}{1-1} - \frac{1}{2}$ is undefined, so use $\frac{1}{1^2} = 1$ Then $\frac{1}{2^2} < \frac{1}{1} - \frac{1}{2}$ $\frac{1}{3^2} < \frac{1}{2} - \frac{1}{3}$ $\frac{1}{4^2} < \frac{1}{3} - \frac{1}{4}$ Therefore $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots\dots + \frac{1}{n^2}$ $< 1 + \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right)\dots\dots\dots + \left(\frac{1}{n-2} - \frac{1}{n-1}\right) + \left(\frac{1}{n-1} - \frac{1}{n}\right)$ $< 1 + 1 + \left(\frac{1}{2} - \frac{1}{2}\right) + \left(\frac{1}{3} - \frac{1}{3}\right) + \left(\frac{1}{4} - \frac{1}{4}\right)\dots\dots\dots + \left(\frac{1}{n-2} - \frac{1}{n-2}\right) + \left(\frac{1}{n-1} - \frac{1}{n-1}\right) - \frac{1}{n}$ $< 2 + 0 + 0 + 0 + \cdots + 0 + 0 - \frac{1}{n}$ $< 2 - \frac{1}{n}$ As n is a positive integer , $\frac{1}{n} > 0$ and $2 - \frac{1}{n} < 2$ $\therefore 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots\dots + \frac{1}{n^2} < 2$	3 marks for correct proof 2 marks for substantial progress toward correct proof 1 mark for some correct working relevant to proof Note: Initialising a proof by induction for $1/(k+1)^2$, in most cases, did not make any progress towards the solution, and so was awarded zero marks, unless the context of this question's hints and assumptions were explicitly utilised effectively.

13(d)MEX 12-6	$v\frac{dv}{dx} = v^2 - 3$ $\frac{dv}{dx} = \frac{v^2 - 3}{v}$ $\frac{dx}{dv} = \frac{v}{v^2 - 3}$ $dx = \frac{v}{v^2 - 3}dv$ $\int dx = \int \frac{v}{v^2 - 3}dv$ $\therefore x = \frac{1}{2}\ln\left\|v^2 - 3\right\| + C$ Given $v = -2$ where $x = 1$ then $C = 1$ $\therefore x = \frac{1}{2}\ln\left\|v^2 - 3\right\| + 1$ $2(x-1) = \ln\left\|v^2 - 3\right\|$	4 Marks: Correct solution. 3 Marks: Makes almost complete progress. 2 Marks: Successful integration 1 Mark: Some relevant progress.

	$$\|v^2 - 3\| = e^{2(x-1)}$$ $\therefore v^2 - 3 = e^{2(x-1)}$ is sufficient for this set of conditions. $$\therefore v = -\sqrt{3 + e^{2(x-1)}}$$ Only the negative square root is relevant due to the particle's initial conditions.	

Question 14

(a)MEX 12-2	(i) If k is even, i.e $k = 2x$, then $k^2 + k = (2x)^2 + 2x$ $= 4x^2 + 2x$ $= 2(2x^2 + x)$ $= 2m$ If k is odd, i.e $k = 2x + 1$, then $k^2 + k = (2x+1)^2 + 2x + 1$ $= 4x^2 + 4x + 1 + 2x + 1$ $= 4x^2 + 6x + 2$ $= 2(2x^2 + 3x + 1)$ $= 2m$	2 marks for showing both results 1 mark for proving only one, or equivalent merit

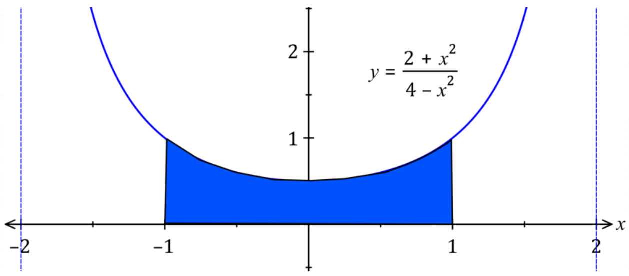
	(ii) Show that the statement is true for $n = 1$ Assume that $n^3 + 5n$ is divisible by 6 for $n = k$ i.e $k^3 + 5k = 6p$ where p is an integer. Now when $n = k + 1$ $(k+1)^3 + 5(k+1) = k^3 + 3k^2 + 3k + 1 + 5k + 5$ $= k^3 + 5k + 3k^2 + 3k + 6$ $= 6p + 3k^2 + 3k + 6$ $= 6p + 6 + 3(k^2 + k)$ * from i) above $= 6p + 6 + 3(2m)$* $= 6p + 6 + 6m$ $= 6(p + m + 1)$ $(k+1)^3 + 5(k+1)$ is divisible by 6 $\therefore$ if true for $n = k$, then also true for $n = k + 1$, but since true for $n = 1$, by induction is true for all integral values, $n \geq 1$	3 marks for correct and complete proof 2 marks for substantial progress in proof with either an error or incomplete statements or equivalent merit 1 mark for initial working relevant to the proof or equivalent merit

(b) MEX 12-2	Prove that $\frac{a+b}{2} \geq \sqrt{ab}$ if $a, b \geq 0$ We know that: $\left(\sqrt{a}-\sqrt{b}\right)^2 \geq 0$ since a,b are real. $i.e. \sqrt{a}^2 - 2\sqrt{a}\sqrt{b} + \sqrt{b}^2 \geq 0$ $a+b \geq 2\sqrt{ab}$ $\frac{a+b}{2} \geq \sqrt{ab}$ as required.	2 marks for correct and complete proof 1 mark for significant working toward proof
(c) MEX 12-3	(i) Point $(-1,2,-3)$ has position vector $\begin{pmatrix}-1\\2\\-3\end{pmatrix}$ Direction vector = $\begin{pmatrix}1\\-2\\2\end{pmatrix}$ Therefore $\underset{\sim}{r_1} = \begin{pmatrix}-1\\2\\-3\end{pmatrix} + s\begin{pmatrix}1\\-2\\2\end{pmatrix}$	1 mark: Correct answer.
	(ii) To be parallel, the lines need the same direction vector, but must not coincide. i.e. the given answer must not pass through point (1, -2, 6). e.g answer: the vector: $\begin{pmatrix}-1\\2\\3\end{pmatrix}$ is parallel to line l_2	1 mark: Correct answer.
	(iii) Equate components of $\boldsymbol{r_1}$, $\boldsymbol{r_2}$ gives: $s-1=-t+1........(1)$ $-2s+2=2t-2...(2)$ $2s-3=3t+6......(3)$ Solving gives $s=3 \quad t=-1$ These 2 solutions should be tested against **<u>all three</u>** equations to prove that the two lines intersect in 3D. Then substituting into the *LHS* and the *RHS* of the above will both give the point of intersection $(2,-4,3)$ Using *s* or *t* must result in the same outcome.	3 marks: Equates components, evaluates parameters and finds the point of intersection. 2 marks: Major progress towards solution. 1 mark: Some relevant progress. Note: This marking scheme could be adopted more rigorously if it was aimed at a higher band of candidate. So, even if you got 3 marks this time, there is a possibility that the HSC marking guideline could require all 3 equations tested, instead of just 2.
	(iv) $\cos\theta = \dfrac{\begin{pmatrix}1\\-2\\2\end{pmatrix} \bullet \begin{pmatrix}-1\\2\\3\end{pmatrix}}{\sqrt{1^2+(-2)^2+2^2}\sqrt{(-1)^2+2^2+3^2}}$ $= \dfrac{1}{3\sqrt{14}}$ $\therefore \theta = 84 \cdot 9°$	3 marks: Correct solution. 2 marks: Progress regarding both the dot product and the lengths of the directon vectors. 1 mark: Some relevant progress.

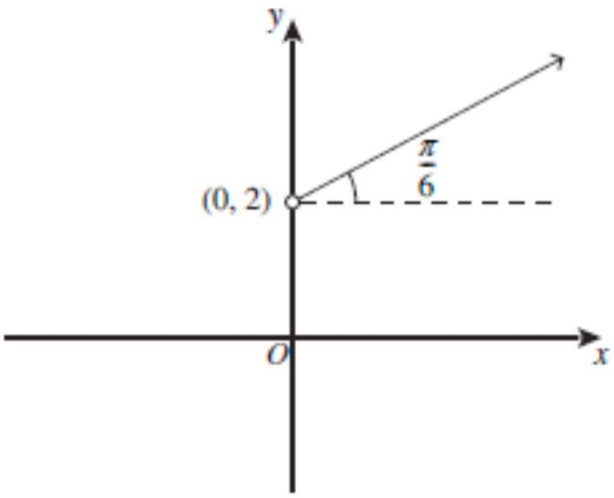
Year 12	Mathematics Extension 2	Assess. Task 4 2020 HSC
Question No. 15	Solutions and Marking Guidelines	

Part / Outcome	**Solutions**	**Marking Guidelines**

(a)(i) **MEX12-4**	$(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$ *and* $(\cos\theta + i\sin\theta)^5 = (\cos\theta)^5 + 5(\cos\theta)^4(i\sin\theta) + 10(\cos\theta)^3(i\sin\theta)^2$ $+10(\cos\theta)^2(i\sin\theta)^3 + 5(\cos\theta)(i\sin\theta)^4 + (i\sin\theta)^5$ Equating reals: $\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ Equating Imaginaries: $\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$	<u>2 marks</u>: correct solution <u>1 mark</u>: substantially correct solution
(a)(ii) **MEX12-4**	$\tan 5\theta = \frac{\sin 5\theta}{\cos 5\theta}$ $= \frac{5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta}{\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta}$ *divide through by* $\cos^5\theta$ $= \frac{5t - 10t^3 + t^5}{1 - 10t^2 + 5t^4}$, where $t = \tan\theta$	<u>1 mark</u>: correct solution (dividing to get tan5θ)
(a)(iii) **MEX12-4**	if $\tan 5\theta = 0$ then $5\theta = 0, \pi, 2\pi, 3\pi, 4\pi...$ $\theta = 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}...$ Also, if $\tan 5\theta = 0$ then $\frac{5t - 10t^3 + t^5}{1 - 10t^2 + 5t^4} = 0$ ie $5t - 10t^3 + t^5 = 0$ $t(t^4 - 10t^2 + 5) = 0$ and roots of $t^4 - 10t^2 + 5 = 0$ must be $t = \tan\frac{\pi}{5}, \tan\frac{2\pi}{5}, \tan\frac{3\pi}{5}, \tan\frac{4\pi}{5}$	<u>3 marks</u>: correct solution <u>2 marks</u>: substantially correct solution <u>1 mark</u>: significant progress towards correct solution Note: The product of roots must match the equation in your solution for full marks. Simply stating product of roots = 5 from an unknown (or unshown) equation is not enough

(b)(i) **MEX12-5**	product of roots, $\alpha\beta\gamma\delta = \frac{e}{a}$ $\tan\frac{\pi}{5}\tan\frac{2\pi}{5}\tan\frac{3\pi}{5}\tan\frac{5\pi}{5} = 5$	
(b)(ii) **MEX12-5**	$f(x) = \frac{2+x^2}{4-x^2}$ $= \frac{-(4-x^2)+6}{4-x^2}$ $= -1 + \frac{6}{4-x^2}$  $A = \int_{-1}^{1}\left(-1+\frac{6}{4-x^2}\right)dx$ $= 2\int_{0}^{1}\left(-1+\frac{6}{4-x^2}\right)dx$ Using partial fractions: $\frac{6}{4-x^2} = \frac{A}{2-x} + \frac{B}{2+x}$ $6 = A(2+x) + B(2-x)$ $\Rightarrow A = \frac{3}{2}, B = \frac{3}{2}$	<u>1 mark</u>: correct solution <u>3 marks</u>: correct solution <u>2 marks</u>: substantially correct solution <u>1 mark</u>: significant progress towards correct solution

(c) MEX12-4	$A=2\int_0^1\left(-1+\frac{3}{2}.\frac{1}{2-x}+\frac{3}{2}.\frac{1}{2+x}\right)dx$ $=2\left[-x-\frac{3}{2}\ln(2-x)+\frac{3}{2}\ln(2+x)\right]_0^1$ $=\left[-2x-3\ln(2-x)+3\ln(2+x)\right]_0^1$ $=\left[-2x+3\ln\left(\frac{2+x}{2-x}\right)\right]_0^1$ $=\left[-2+3\ln(3)\right]-\left[0-3\ln(1)\right]$ $=3\ln 3-2$	
(d)(i) MEX12-4	Let $u=a+2i$ and $v=1+ib$ $u+v=(a+1)+(b+2)i \qquad ...(1)$ $-uv=(-a+2b)-(ab+2)i \qquad ...(2)$ Equating imaginary parts of (1) and (2) $b+2=-ab-2$ $ab+b+4=0 \qquad ...(3)$ Equating real parts of (1) and (2) $a+1=-a+2b$ $2a=2b-1$ $a=\frac{2b-1}{2} \qquad ...(4)$ sub $(3)\rightarrow(4)$ $\frac{1}{2}(2b-1)+b+4=0$ $2b^2+b+8=0$ $D=1-4.2.8$ $=-63<0$ no real solutions But *a*, *b* are real, $\therefore$ no solution	<u>2 marks</u>: correct solution <u>1 mark</u>: substantially correct solution

(d)(ii) **MEX12-4**	$\arg(z-2i)=\frac{\pi}{6}$ $\arg(x+iy-2i)=\frac{\pi}{6}$ $\arg(x+(y-2)i)=\frac{\pi}{6}$ $\frac{y-2}{x}=\tan\frac{\pi}{6}$ $y-2=\frac{1}{\sqrt{3}}x$ $y=\frac{1}{\sqrt{3}}x+2,\ x>0$  	<u>2 marks</u> : correct solution <u>1 mark</u> : substantially correct solution Note: $x > 0$ should have been stated, but did not cost any marks in this instance. *However*, missing this concept did have an impact on part (ii) – see comment below <u>1 mark</u>: correct solution Note: this is a standard example, and the open circle at (0,2) is a very important detail which was required for the mark. For *z* to have an argument, it must lie beyond that point.

	12(c) solution for the *correct* question Let $u = u_1 + 2i$ and $v = -1 + v_2 i$. $u + v = (u_1 - 1) + (2 + v_2)i \qquad (1)$ $-uv = (u_1 + 2v_2) + (2 - u_1 v_2)i \qquad (2)$ Equating the real components of (1) and (2) gives $u_1 - 1 = u_1 + 2v_2 \Rightarrow v_2 = -\frac{1}{2}$. Equating the imaginary components of (1) and (2) with $v_2 = -\frac{1}{2}$ gives $-\frac{1}{2} = \frac{1}{2}u_1 \Rightarrow u_1 = -1$. So the two complex numbers are $u = -1 + 2i$ and $v = -1 - \frac{1}{2}i$.	

Year 12 (task 4)	Extension 2 Mathematics	2020 Trial HSC Examination
Question No. 16	**Solutions and Marking Guidelines**	

HSC Out-come	Solutions	Marking Guidelines
MEX12-3	Question 16 (a) If $\overrightarrow{BA}.\overrightarrow{BC}=0$ then $\angle ABC$ is $90°$ Required to show that $\overrightarrow{BA}.\overrightarrow{BC}=0$: $LHS=\overrightarrow{BA}.\overrightarrow{BC}$ $=\left[\begin{pmatrix}1\\0\\5\end{pmatrix}-\begin{pmatrix}-1\\2\\4\end{pmatrix}\right]\bullet\left[\begin{pmatrix}3\\5\\2\end{pmatrix}-\begin{pmatrix}-1\\2\\4\end{pmatrix}\right]$ $=\begin{pmatrix}2\\-2\\1\end{pmatrix}\bullet\begin{pmatrix}4\\3\\-2\end{pmatrix}$ $=8-6-2$ $=0$ $=RHS$ $\therefore \angle ABC$ is $90°$ since $\overrightarrow{BA}.\overrightarrow{BC}=0$, hence ΔABC is a right angled triangle.	Award 2 ~Complete correct solution Award 1 ~Significant progress towards correct solution

MEX12-2

Question 16 (b) First solution

Required to prove that $33^n - 16^n - 28^n + 11^n$ is divisible by 85 for $n \geq 2,\ n \in \mathbb{Z}^+$

Using,

$$a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + \ldots + ab^{n-2} + b^{n-1}),\ \text{for } n \geq 2,\ n \in \mathbb{Z}^+$$

then

$$33^n - 16^n = (33-16)(33^{n-1} + 33^{n-2} \times 16 + \ldots + 33 \times 16^{n-2} + 16^{n-1}),\ \text{for } n \geq 2,\ n \in \mathbb{Z}^+$$
$$= 17A \qquad \text{where } A = 33^{n-1} + 33^{n-2} \times 16 + \ldots + 33 \times 16^{n-2} + 16^{n-1},\ n \geq 2,\ n \in \mathbb{Z}^+$$

and

$$11^n - 28^n = (11-28)(11^{n-1} + 11^{n-2} \times 28 + \ldots + 11 \times 28^{n-2} + 28^{n-1}),\ \text{for } n \geq 2,\ n \in \mathbb{Z}^+$$
$$= -17B \qquad \text{where } B = 11^{n-1} + 11^{n-2} \times 28 + \ldots + 11 \times 28^{n-2} + 28^{n-1},\ n \geq 2,\ n \in \mathbb{Z}^+$$

Also,

$$33^n - 28^n = (33-28)(33^{n-1} + 33^{n-2} \times 28 + \ldots + 33 \times 28^{n-2} + 28^{n-1}),\ \text{for } n \geq 2,\ n \in \mathbb{Z}^+$$
$$= 5C \qquad \text{where } C = 33^{n-1} + 33^{n-2} \times 28 + \ldots + 33 \times 28^{n-2} + 28^{n-1},\ n \geq 2,\ n \in \mathbb{Z}^+$$

and

$$11^n - 16^n = (11-16)(11^{n-1} + 11^{n-2} \times 16 + \ldots + 11 \times 16^{n-2} + 16^{n-1}),\ \text{for } n \geq 2,\ n \in \mathbb{Z}^+$$
$$= -5D \qquad \text{where } D = 11^{n-1} + 11^{n-2} \times 16 + \ldots + 11 \times 16^{n-2} + 16^{n-1},\ n \geq 2,\ n \in \mathbb{Z}^+$$

so now,

$$33^n - 16^n - 28^n + 11^n = 17A - 17B$$
$$= 17(A - B) \quad \text{where } A = 33^{n-1} + 33^{n-2} \times 16 + \ldots + 33 \times 16^{n-2} + 16^{n-1},\ n \geq 2,\ n \in \mathbb{Z}^+$$
$$\text{where } B = 11^{n-1} + 11^{n-2} \times 28 + \ldots + 11 \times 28^{n-2} + 28^{n-1},\ n \geq 2,\ n \in \mathbb{Z}^+$$

and

$$33^n - 16^n - 28^n + 11^n = 15C - 15D$$
$$= 5(C - D) \quad \text{where } C = 33^{n-1} + 33^{n-2} \times 28 + \ldots + 33 \times 28^{n-2} + 28^{n-1},\ n \geq 2,\ n \in \mathbb{Z}^+$$
$$\text{where } D = 11^{n-1} + 11^{n-2} \times 16 + \ldots + 11 \times 16^{n-2} + 16^{n-1},\ n \geq 2,\ n \in \mathbb{Z}^+$$

$\therefore 33^n - 16^n - 28^n + 11^n$ is divisible by 17 and 5, for $n \geq 2,\ n \in \mathbb{Z}^+$.

Since 85 has prime factors of 17 and 5 then $33^n - 16^n - 28^n + 11^n$ is divisible by 85, for $n \geq 2,\ n \in \mathbb{Z}^+$.

Award 3

~Complete correct solution

Award 2

~Significant progress towards correct solution

Award 1

~Limited progress towards correct solution

Question 16

(b) Second solution using Proof by Mathematical induction.

Required to prove that $33^n - 16^n - 28^n + 11^n$ is divisible by 85 for $n \geq 2,\ n \in \mathbb{Z}^+$

i.e. $33^n - 16^n - 28^n + 11^n = 85M, \quad M \in \mathbb{Z}$

Prove true for $n = 2$,

$$\begin{aligned} LHS &= 33^2 - 16^2 - 28^2 + 11^2 \\ &= 170 \\ &= 85 \times 2 \\ &= 85M, \text{ where } M = 2 \\ &= RHS \end{aligned}$$

$\therefore$ True for $n = 2$

Assume true for $n = k,\ \ k \geq 2$ and $k \in \mathbb{Z}^+$

i.e. $33^k - 16^k - 28^k + 11^k = 85M,\ \ M \in \mathbb{Z}$ ....(1)

Prove true for $n = k+1,\ \ k \geq 2$ and $k \in \mathbb{Z}^+$

i.e. $33^{k+1} - 16^{k+1} - 28^{k+1} + 11^{k+1} = 85P,\ \ P \in \mathbb{Z},\ \ k \geq 2$ and $k \in \mathbb{Z}^+$

$$\begin{aligned} LHS &= 33^{k+1} - 16^{k+1} - 28^{k+1} + 11^{k+1} \\ &= 33\left(33^k\right) - 16\left(16^k\right) - 28\left(28^k\right) + 11\left(11^k\right) \\ &= 33\left(85M + 16^k + 28^k - 11^k\right) - 16\left(16^k\right) - 28\left(28^k\right) + 11\left(11^k\right) \text{ using assumption (1)} \\ &= 85(33M) + 33\left(16^k\right) + 33\left(28^k\right) - 33\left(11^k\right) - 16\left(16^k\right) - 28\left(28^k\right) + 11\left(11^k\right) \\ &= 85(33M) + (33-16)\left(16^k\right) + (33-28)\left(28^k\right) + (11-33)\left(11^k\right) \\ &= 85(33M) + 17\left(16^k\right) + 5\left(28^k\right) - 22\left(11^k\right) \end{aligned}$$

Now using mathematical induction again prove that $17\left(16^n\right) + 5\left(28^n\right) - 22\left(11^n\right)$ is divisible by 85 for $n \geq 2,\ n \in \mathbb{Z}^+$

i.e. $17\left(16^n\right) + 5\left(28^n\right) - 22\left(11^n\right) = 85Q, \quad Q \in \mathbb{Z},\ \ n \geq 2,\ \ n \in \mathbb{Z}^+$

Prove true for $n = 2$

$$\begin{aligned} LHS &= 17\left(16^2\right) + 5\left(28^2\right) - 22\left(11^2\right) \\ &= 5610 \\ &= 85 \times 66 \\ &= 85Q, \quad \text{where } Q = 66 \\ &= RHS \end{aligned}$$

$\therefore$ True for $n = 2$.

Award 3

~Complete correct solution

Award 2

~Significant progress towards correct solution

Award 1

~Limited progress towards correct solution

MEX12-2

	Assume true for $n = k$ i.e. $17(16^k)+5(28^k)-22(11^k)=85Q, \quad Q\in\mathbb{Z},\ k\geq 2,\ k\in\mathbb{Z}^+$.........................(2) Prove true for $n=k+1$ i.e. $17(16^{k+1})+5(28^{k+1})-22(11^{k+1})=85R,$ where $R\in\mathbb{Z}, k\geq 2$ and $k\in\mathbb{Z}^+$ $LHS = 17(16^{k+1})+5(28^{k+1})-22(11^{k+1})$ $=17(16)(16^k)+5(28)(28^k)-22(11)(11^k)$ $=(16)(17(16^k))+140(28^k)-242(11^k)$ $=(16)(85Q-5(28^k)+22(11^k))+140(28^k)-242(11^k)$ using assumption (2) $=85(16Q)-80(28^k)+352(11^k)+140(28^k)-242(11^k)$ $=85(16Q)+60(28^k)+110(11^k)$ Now using mathematical induction again prove that $60(28^n)+110(11^n)$ is divisible by 85 for $n\geq 2,\ n\in\mathbb{Z}^+$ i.e. $60(28^n)+110(11^n)=85B, \quad B\in\mathbb{Z}, n\geq 2,\ n\in\mathbb{Z}^+$ Prove true for $n=2$ $LHS = 60(28^2)+110(11^2)$ $=60350$ $=85\times 710$ $=85B,$ where $B=710$ $=RHS$ $\therefore$ True for $n=2$. Assume true for $n=k,\ \ k\geq 2,\ k\in\mathbb{Z}^+$ i.e. $60(28^k)+110(11^k)=85B, \quad B\in\mathbb{Z},\ k\geq 2,\ k\in\mathbb{Z}^+$.....................(3) Prove true for $n=k+1$ i.e. $60(28^{k+1})+110(11^{k+1})=85A, \quad A\in\mathbb{Z},\ k\geq 2,\ k\in\mathbb{Z}^+$ $LHS = 60(28^{k+1})+110(11^{k+1})$ $=60(28)(28^k)+110(11)(11^k)$ $=28(60(28^k))+110(11)(11^k)$ $=28(85B-110(11^k))+110(11)(11^k)$ using assumption (3) $=85(28B)-3080(11^k)+1210(11^k)$ $=85(28B)-1870(11^k)$ $=85(28B-22(11^k))$ $=85A,$ where $A=28B-22(11^k),\ A\in\mathbb{Z},\ k\geq 2,\ k\in\mathbb{Z}^+$ $\therefore 60(28^{k+1})+110(11^{k+1})=85A,$ and the statement is true for $n=k+1,\ k\geq 2,\ k\in\mathbb{Z}^+$ $\therefore$ As the statement is true for $n=2,\ n=k$ and $n=k+1$, then by mathematical induction it is proven that $60(28^n)+110(11^n)$ is divisible by 85 for $\forall n\in\mathbb{Z}^+, n\geq 2$.	

Hence it follows that

$85\left(16Q\right)+60\left(28^k\right)+110\left(11^k\right)=85(16Q)+85A, \quad Q\in\mathbb{Z},\ A\in\mathbb{Z},\ k\geq 2,\ k\in\mathbb{Z}^+$

$=85(16Q+A)$

$=85R, \quad \text{where } R=16Q+A,\ R\in\mathbb{Z}$

$\therefore 17\left(16^{k+1}\right)+5\left(28^{k+1}\right)-22\left(11^{k+1}\right)=85R,$ and the statement is true for $n=k+1,\ k\geq 2,\ k\in\mathbb{Z}^+$.

$\therefore$ As the statement is true for $n=2,\ n=k$ and $n=k+1$, by mathematical induction it is proven that $17\left(16^n\right)+5\left(28^n\right)-22\left(11^n\right)$ is divisible by 85 $\forall n\in\mathbb{Z}^+,\ n\geq 2$.

Hence it also follows that

$85\left(33M\right)+17\left(16^k\right)+5\left(28^k\right)-22\left(11^k\right)=85(33M)+85Q, \quad M\in\mathbb{Z},\ Q\in\mathbb{Z},\ k\geq 2,\ k\in\mathbb{Z}^+$

$=85(33M+Q), \quad M\in\mathbb{Z}, Q\in\mathbb{Z},\ k\geq 2,\ k\in\mathbb{Z}^+$

$=85P \quad P\in\mathbb{Z},\ k\geq 2,\ k\in\mathbb{Z}^+$

$\therefore 33^{k+1}-16^{k+1}-28^{k+1}+11^{k+1}=85P$ and the statement is true for $n=k+1,\ k\geq 2,\ k\in\mathbb{Z}^+$.

$\therefore$ As the statement is true for $n=2,\ n=k$ and $n=k+1$, by mathematical induction it is proven that $33^n-16^n-28^n+11^n$ is divisible by 85 $\forall n\in\mathbb{Z}^+,\ n\geq 2$.

Question 16

(c) First solution

$\overrightarrow{PR}=\left(9i+3j+8k\right)-\left(i+j+k\right)$

$=8i+2j+7k$

$\overrightarrow{PQ}=\frac{2}{5}\left(8i+2j+7k\right)$

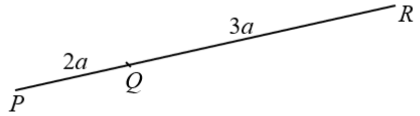
$\overrightarrow{OQ}=\overrightarrow{OP}+\overrightarrow{PQ}$

$=\left(i+j+k\right)+\frac{2}{5}\left(8i+2j+7k\right)$

$=\frac{21}{5}i+\frac{9}{5}j+\frac{19}{5}k$

$\therefore\ Q=\begin{pmatrix}\frac{21}{5}\\ \frac{9}{5}\\ \frac{19}{5}\end{pmatrix}$ or $Q=\frac{21}{5}i+\frac{9}{5}j+\frac{19}{5}k.$

(c) Second solution



	let $Q = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $\frac{\overrightarrow{PQ}}{\overrightarrow{QR}} = \frac{2}{3}$ $\therefore \overrightarrow{PQ} = \frac{2}{3}\overrightarrow{QR}$ $\begin{pmatrix} x-1 \\ y-1 \\ z-1 \end{pmatrix} = \frac{2}{3}\begin{pmatrix} 9-x \\ 3-y \\ 8-z \end{pmatrix}$ $\therefore x-1 = \frac{2}{3}(9-x)$ $3x-3 = 18-2x$ $5x = 21$ $x = \frac{21}{5}$ $\therefore y-1 = \frac{2}{3}(3-y)$ $3y-3 = 6-2y$ $5y = 9$ $\text{y} = \frac{9}{5}$ $\therefore z-1 = \frac{2}{3}(8-z)$ $3z-3 = 16-2z$ $5z = 19$ $\text{z} = \frac{19}{5}$ $\therefore\ Q = \begin{pmatrix} \frac{21}{5} \\ \frac{9}{5} \\ \frac{19}{5} \end{pmatrix}$ or $Q = \frac{21}{5}i + \frac{9}{5}j + \frac{19}{5}k.$	

	Question 16 $(d)(i)$ $I_n = \int_0^1 x^n \tan^{-1} x \, dx$ let $u = \tan^{-1} x \qquad \frac{dv}{dx} = x^n$ $\frac{du}{dx} = \frac{1}{1+x^2} \qquad v = \frac{x^{n+1}}{n+1}$ $I_n = \left[\frac{x^{n+1}}{n+1} . \tan^{-1} x \right]_0^1 - \int_0^1 \frac{x^{n+1}}{(n+1)(1+x^2)} \, dx$ $= \frac{1}{n+1} . \frac{\pi}{4} - \frac{1}{n+1} \int_0^1 \frac{x^{n+1}}{(1+x^2)} \, dx$ $= \frac{1}{n+1} \left(\frac{\pi}{4} - \int_0^1 \frac{x^{n+1}}{(1+x^2)} \, dx \right)$ $(n+1) I_n = \frac{\pi}{4} - \int_0^1 \frac{x^{n+1}}{(1+x^2)} \, dx$	Award 3 ~Complete correct solution Award 2 ~Significant progress towards correct solution Award 1 ~Limited progress towards correct solution
MEX12-3	Question 16 (d)(ii) $(n+1) I_n = \frac{\pi}{4} - \int_0^1 \frac{x^{n+1}}{(1+x^2)} \, dx$ from (i) let $n = 0$ $(0+1) I_0 = \frac{\pi}{4} - \int_0^1 \frac{x^{0+1}}{(1+x^2)} \, dx$ $I_0 = \frac{\pi}{4} - \int_0^1 \frac{x}{(1+x^2)} \, dx$ $= \frac{\pi}{4} - \left[\frac{1}{2} \ln(1+x^2) \right]$ $= \frac{\pi}{4} - \frac{1}{2} (\ln 2 - \ln 0)$ $= \frac{\pi}{4} - \frac{1}{2} \ln 2$	Award 3 ~Complete correct solution Award 2 ~Significant progress towards correct solution Award 1 ~Limited progress towards correct solution

MEX12-3	Question 16 (d)(iii) $(n+1)I_n = \frac{\pi}{4} - \int_0^1 \frac{x^{n+1}}{(1+x^2)}\,dx$ from part(i)..............(A) let $n = n+2$ $(n+2+1)I_{n+2} = \frac{\pi}{4} - \int_0^1 \frac{x^{n+2+1}}{(1+x^2)}\,dx$ $(n+3)I_{n+2} = \frac{\pi}{4} - \int_0^1 \frac{x^{n+3}}{(1+x^2)}\,dx$ ....(B) (A)+(B) gives: $(n+1)I_n + (n+3)I_{n+2} = \frac{\pi}{4} - \int_0^1 \frac{x^{n+1}}{(1+x^2)}\,dx + \frac{\pi}{4} - \int_0^1 \frac{x^{n+3}}{(1+x^2)}\,dx$ $= \frac{\pi}{2} - \int_0^1 \frac{x^{n+1}+x^{n+3}}{(1+x^2)}\,dx$ $= \frac{\pi}{2} - \int_0^1 \frac{x^{n+1}(1+x^2)}{(1+x^2)}\,dx$ $= \frac{\pi}{2} - \int_0^1 x^{n+1}\,dx$ $= \frac{\pi}{2} - \left[\frac{x^{n+1}}{n+2}\right]_0^1$ $= \frac{\pi}{2} - \frac{1}{n+2}$	Award 2 ~Complete correct solution Award 1 ~Significant progress towards correct solution

MEX12-5	Question 16 (d)(iv) from part (iii) $(n+1)I_n+(n+3)I_{n+2}=\frac{\pi}{2}-\frac{1}{n+2}$ let $n=2$ $(2+1)I_2+(2+3)I_{2+2}=\frac{\pi}{2}-\frac{1}{2+2}$ $3I_2+5I_4=\frac{\pi}{2}-\frac{1}{4}$..........................(A) let $n=0$ $(0+1)I_0+(0+3)I_{0+2}=\frac{\pi}{2}-\frac{1}{0+2}$ $I_0+3I_2=\frac{\pi}{2}-\frac{1}{2}$ ....(B) from part (ii) $I_0=\frac{\pi}{4}-\frac{1}{2}\ln 2$.....................$(C)$ Substitute (C) into (B) $\therefore \quad \frac{\pi}{4}-\frac{1}{2}\ln 2+3I_2=\frac{\pi}{2}-\frac{1}{2}$ $3I_2=\frac{\pi}{2}-\frac{1}{2}-\frac{\pi}{4}+\frac{1}{2}\ln 2$ $3I_2=\frac{\pi}{4}-\frac{1}{2}+\frac{1}{2}\ln 2$ ....(D) Now, Substitute (B) into (A) $\frac{\pi}{4}-\frac{1}{2}+\frac{1}{2}\ln 2+5I_4=\frac{\pi}{2}-\frac{1}{4}$..........................(A) $5I_4=\frac{\pi}{2}-\frac{1}{4}-\frac{\pi}{4}+\frac{1}{2}-\frac{1}{2}\ln 2$ $5I_4=\frac{\pi}{4}+\frac{1}{4}-\frac{1}{2}\ln 2$ $I_4=\frac{1}{5}\left(\frac{\pi}{4}+\frac{1}{4}-\frac{1}{2}\ln 2\right)$	Award 2 ~Complete correct solution Award 1 ~Significant progress towards correct solution